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MATHEMATICS STUDENTS' CONCEPTIONS OF MATHEMATICS

A. REID, P. PETOCZ, G. SMITH, L. WOOD AND E. DORTINS

Abstract. In this paper we report on the first phase of a study of mathematics students' conceptions of mathematics, their notions of professional work in the mathematical sciences, and the relationship between these ideas and the way they learn mathematics. Interviews were carried out with 22 later-year students majoring in the mathematical sciences, investigating their views of their subject and future profession. Responses were analysed using a phenomenographic approach. We found that students' conceptions of mathematics could be arranged in a hierarchy ranging from the narrowest view as a focus on components, through a notion of mathematics as a focus on models, to the broadest view of mathematics as an approach to life and a way of thinking. The results highlight areas that are important to the development of students' mathematical thinking and their future professional role as mathematical scientists.

1. Background

As mathematicians and mathematics educators, we need no convincing of the beauty, interest or utility of our subject. At a national or international level, scientific institutions and governments seem to recognise its importance. For instance, a recent report [**5**] asserts:

> The mathematical sciences are critical to Australia's economic competitiveness and quality of life, and will become more so. The mathematical sciences are generic and enabling technologies. They are essential to the prosperity of many value-adding industries in Australia. ... Individuals and groups pursuing research in the mathematical sciences constitute a human and intellectual resource of the utmost national significance. (Executive summary, p. x)

However, researchers in mathematics education worldwide (such as [**3**]) have identified a serious problem: despite the importance of the mathematical sciences, and despite the opportunities and remuneration available to graduates of mathematics, fewer students are enrolling for degrees in mathematics and many of those that do enrol don't seem to understand the nature of mathematics itself or professional work as a mathematician!

The job of "mathematician" is not obvious, visible or well defined. It encompasses a wide range of careers that are related through the tools and techniques of mathematics. For most students in particular, the nature of mathematical work is not at all clear, and hence it is not easy for them to make a connection between

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Key words and phrases: Mathematics education, conceptions, professional formation, phenomenography.

what they are learning at university and what they will be doing as a mathematician. Many studies have investigated the step from school to university mathematics study [**19**], but the transition from university study to professional work is less well understood. The project that we report on here investigates students' understanding of learning mathematics, their perception of professional work in the mathematical sciences and the link between them. We identify the important difference between simply learning mathematics and becoming a professional mathematician, and highlight areas that are important to the development of students' mathematical thinking and their future profession. On the basis of our previous work and our results to date, we hypothesise that students who have a clear understanding of their future professional capabilities and needs will engage more deeply with the subject matter.

Schoenfeld [**12**] distinguishes two types of research in mathematics education: one with immediate goals, the other aimed at extending thinking about a particular phenomenon. He summarises the result of decades of investigations as a meta-finding: "*We don't perceive reality directly, but rather we build mental structures that shape the ways in which we perceive reality.*" Our project falls into Schoenfeld's second type, and focuses on the exploration of "*the ways in which we* [or our students] *perceive reality*". The project builds on our previous investigations of the communication needs of mathematicians [**6, 15, 16**], broader approaches to mathematics learning and assessment [**13, 17, 18**], and our research in students' conceptions of statistics and statistics learning [**7, 10**].

Although there is a large body of research in mathematics education, relatively little has been investigated from the viewpoint of students who plan to be mathematical professionals. Crawford *et al.* [**1**] undertook a study of students' conceptions of mathematics and how it is learned: however, their study was carried out in first-year mathematics classes that contained few students who were planning to become mathematicians. They explored the relation between the students' own perceptions of the discipline and their learning. They distinguished between fragmented and cohesive conceptions of mathematics, and found a correlation between students' conception of the subject and their approach to learning in the subject. Those with a fragmented conception were likely to use a surface approach to their study, while those with a cohesive conception were more likely to use a deep approach. Reid [**8**] identified a close relationship between students' perception of work and their ideas of and approaches to learning. Her initial findings were generalised from music to other professional areas as the *Professional Entity* (see [**10**]). In a related area where students have a clearer understanding of the nature of their work, we have shown that there is a wide range of conceptions of statistics, statistical work, and learning in statistics [**7, 10**]. The narrowest views of *statistics* were as isolated technical components, while the narrowest conception of *learning* in statistics was of doing isolated activities required in order to pass the subject. The broadest conception of *statistics* was that of an inclusive tool to solve problems, whilst the broadest conception of *learning* in statistics was that it is a way of personally interpreting and understanding the world. Studies in other areas (see [**7**]) have found that some students use higher, integrated conceptual levels in their approach to learning, while others use lower, fragmented levels. Importantly, students who identify with the lower levels can be encouraged to engage with their learning at a higher level through an appropriate learning environment [**9**].

Our project seeks to integrate the research on work-based learning with research on mathematics education within an institution. Although this approach is not commonly used in tertiary mathematics education, students' experience or perceptions of work frame their university experiences, and there is evidence to indicate that this relation enhances the quality of learning. We suggest that this aspect of our research can inform the development of curriculum in mathematics where 'virtual experiences' of work will "*give relevance to the course, and ... better prepare students for professional practice*" [**14**, p. 152].

2. Method

We are carrying out an extended research project investigating mathematics students' ideas about mathematics and working as professionals in the mathematical sciences, and the impact that these ideas have on their learning of mathematics. An important and innovative aspect of our project is the development of an informed theoretical framework for a total learning environment that focuses on *students' experiences* rather than on the artefacts of their learning. We believe that this will provide a theoretical framework to inform the development of future curriculum in mathematics and address the problems of declining enrolments and interest in the field of mathematics referred to earlier.

The project consists of several phases, and here we report on the results from the first phase—a series of in-depth interviews with 22 second to fourth-year students majoring in an area of the mathematical sciences (statistics, mathematical finance, operations research). Students in relevant classes were invited to participate, were informed of the research questions and aims, and interviewed by one of us not involved in teaching them (Dortins): we would like to acknowledge the help of our participating students. The transcripts (over 90,000 words) formed the raw material for our study.

To analyse the data, we used the approach of phenomenography which looks at how people experience, understand and ascribe meaning to a specific situation or phenomenon [**4**]. It is a qualitative orientation to research that takes a non-dualist perspective and is often used to describe the experience of learning and/or teaching. This means that learning and teaching are seen as a relation between the person and the situation that they are experiencing. The outcome of a phenomenographic study is a hierarchical set of logically related categories, from the narrowest and most limited to the broadest and most inclusive. This is referred to as the *outcome space* for the research. Phenomenography defines aspects that are critically *different* within a group involved in the same situation. It is these differences that make one way of seeing mathematics qualitatively different from another. Thus, the categories are defined by their qualitative difference from the other categories, reported in order of their inclusivity and sophistication. In a phenomenographic study, the questions posed are designed to encourage the participants to think about why they experience the phenomenon in certain ways and how they constitute meaning of the phenomenon. In this case, students responded to the key questions: *What do you think mathematics is about? How do you go about learning mathematics? What do you aim to achieve when you are learning in mathematics? What do you think* *it will be like to work as a qualified mathematician?* The range of questions was designed to focus students' awareness on different aspects related to their experience of mathematics, and included probing questions which responded to their answers.

Using the phenomenographic approach, categories describing the variation in students' conceptions of mathematics were suggested, refined and checked by repeated reading. The final categories were confirmed by identification of appropriate quotes in the transcripts. The quotes that we use have been labelled with pseudonyms. It is important to note, however, that the quotes used are illustrative of the conceptual categories, and that the aim is not to put any particular student into a specific category. Each individual quote is not necessarily indicative of the meaning of the category, but merely supportive, and the richness of each category is defined by the *whole* set of transcripts.

3. Conceptions of Mathematics

Our analysis of the transcripts identified three qualitatively different ways in which students understand mathematics, ranging from limiting to expansive views. We use the term "limiting" to indicate that students who describe such views seemed unable to describe any characteristics of more integrated and expansive views, and this limits their approach to the subject. Such students may only be able to focus their attention within their learning environment on fragmented and isolated aspects of mathematics. Conversely, students who can describe the more integrated and expansive views are able to make use of characteristics throughout the whole range to increase their understanding of mathematics.

(1) Mathematics is about components. In this conception, students see mathematics as made up of individual components. They focus their attention on disparate mathematical activities or aspects of mathematics, including the notion of calculation, interpreted in the widest sense. Students holding this conception responded in this way to the question *What do you think mathematics is about?*:

> *Monique:* Um mathematics it's about, it's not just about calculating and computing, but also um the application that will be useful in the work environment, you know, when we got to work, when we graduate and we find the work, we can use those maths um techniques and, yeah the maths techniques and all those things that we learned in maths we can use that in the work environment. Yeah and yeah, as I said, it's not just about calculating, it's about applying those formulas to practical terms. Yeah. ...
> Um, yeah in my first year I learnt about calculus and, and at first I thought it is just, you know, calculating, but then it has an application to, to say oh, I forgot the example, um we learned some practical issues relating to calculus, yeah. Do you want me to give examples? ...
> Okay, let's just forget about calculus and just get another example. Let's say statistics, yeah it is really useful in the real world, I worked in Bureau of Stats before and I could see how the theory works in practical. Yeah because we had the census and then after they processed the census, they um, after they process it, they, I think they used the stats formulas, like they find the correlation, and those things that relate to statistics and then, yeah and then they use those information to decide, to decide on the um, on the society, on

> the how, for example the budget of the state. Yeah, after they statistically analysed the information, then they will find, they will learn how to allocate um the budget to different states.
>
> *Andy:* Um a set of procedures, a set of, I don't know, a set of I guess methods which we can use to solve problems in the real world, some models we can use to have a look at phenomena or that sort of thing. ...
> I guess numbers and just using them, working with them using these sort of methods, things like regressions or um hypotheses or something to test assumptions you might make about a particular system or something, a company maybe. Yeah it's a bit of a toolbox really, that's about it I think, I can't think of anything else.

Monique's quotes demonstrate the basic features of this conception. She finds difficulty identifying components of her calculus course, aside from the notion that it is about calculating: her statistics example lists the disparate components of a statistical investigation. Andy's quote is more succinct, but demonstrates the same approach.

(2) Mathematics is about models. In this conception, students see mathematics as being about building and using models, translating some aspect of reality into mathematical form. This is qualitatively different from the previous conception as the focus is on the models, rather than the component activities. In some cases, such models are representations of specific situations, such as a production line or a financial process. In other cases, the models being considered are universal principles, such as the law of gravity. Here are some illustrative quotes:

> *George:* Well I sort of, yeah, at the moment my course is sort of directed at my work, so um I'm doing it basically to get an overall understanding of what I do at work. I'm a currency options trader at a bank, and that has a lot of underlying models that they trade off, and that sort of thing, and I want to be able to understand those models and how they are derived basically. That's why I am trying to do this degree. ...
> Well, um basically I work in foreign exchange and I work on the currency options desk which is a derivative of um of an asset price which is the Aussie dollar, so as that varies you get these derivative products which change price, and the prices of these products are sort of based on a certain model which was derived by Black and Scholes and I just want to understand the mathematics that underlie that model, to actually understand my job a lot better, so that's why I am sort of trying to get all the background knowledge through this course.
>
> *Elly:* Okay, I mean if it wasn't for maths we wouldn't have buildings and bridges and things like that, and doing a double major I've just realised, especially with my OR, that I remember my lecturer mentioning something about he was involved in talks um with the, about the Sydney Olympics, um that, er that they were running a simulation on the train timetable, how often trains should come and, you know, there's an expected amount of this many people and stuff, and people don't realise I think that maths is really all around us. I mean people think, oh maths is one plus one, but its not, its really its all around and I don't think that many people realise that, that there are, I mean

> I've just realised now that I'm doing a maths degree as well about how much maths is involved in everyday life.
>
> *Richard:* Okay, I think mathematics is largely about the building of models. I guess, I really should start, well I think it's about abstract thought, which then leads to the building of models, that's my own personal opinion on this. Um and these models can be completely um a thought construct, completely abstract in which one would question perhaps their, at the time at which the model was developed, their practical application to a world, a real world situation. But still, it's the product of someone's intellectual efforts, I think that's a good thing, I like that. Um, on the other, the other direction which I believe mathematics can be looked at is the application or the development of a model which closely aligns itself to a real world situation, with a view to um bringing to someone's enlightenment a solution to a given problem, be it, you know, something to do with the vibration of the mechanics of a car or something like this, so say, okay, this part wears out often, what can we do to fix this? Can we analyse this mathematically? Yes, we can, here are some opportunities for improvement, and, but whichever way you go it is still a model, in the latter description I gave I guess it more closely aligns itself to um someone's particular requirements at the time um, an application. That's my thinking on that one.

These three quotes show students focusing on models. George seems to make an obvious connection with his work and the models that he uses there. Elly discusses models in general, talks about specific models in OR (operations research) and shows her awareness of models around her in everyday life. Richard also looks at mathematics in terms of models, although he sees a distinction between 'pure' abstract models, and models that are 'applied' to the real world.

(3) Mathematics is about life. In this conception, students view mathematics as an approach to life and a way of thinking. They believe that reality can be represented in mathematical terms and their way of thinking about reality is mediated by mathematics. They make a strong personal connection between mathematics and their own lives. This is qualitatively different from the previous conception as the focus is on the broad notion of the relationship between reality and mathematics, rather than on the idea of individual models of aspects of reality. Students holding this conception express it in ways illustrated by the following quotes:

> *Dave:* Um, about a way of thinking about things, a thorough way of thinking about things. ...
> Well I think it gives me a sense of clarity about things, it gives me a bit more confidence that if I'm trying to make a judgement about an issue, a concept, a political debate, anything that you can, that there is a sort of fall-back way of thinking about things that can quite, quite reliably help you form a conclusion, or let you know that a conclusion might not be appropriate. ...
> Ploddy. Taking things step by step and not sort of jumping, not jumping to, its anti-intuitive, I guess that's how I see it, its not intuitive, you can combine the two together, you can have an intuitive idea about something and think that seems to be a sort of natural instinctive way that you might look on something, or a natural type of decision that you might take, and then you

can, using your sort of mathematical training, you could sort of deconstruct that thought and take it apart and think, well is this, is this an appropriate way of thinking, an appropriate way of deciding about something?, and you can be a little bit surprised now and again.

Vitali: Mathematics is describing the world in front of us, just in a different language, the same thing as we have words, spellings, sayings, it's for something else, I mean like vocabulary of something else, within and without our cosmology. Mathematics is great. Mathematics is a set of thinking and it's something that you can develop in your life. ... I think mathematics, mathematics, everybody has to go through that, even if you're, what's the, you are not able to, I don't know, discover something new or formulas or anything, you have to just go through the course to start your mind thinking and just ability to look at the things and just basically solve them, that's what mathematics is.

Eddie: Oh jeez, um, um, oh I don't know, that's a very hard question. Um describing the real world I suppose, um yeah, I mean that would be my answer.

...

Um, you know, when we wanted to predict when the, you know, when the seasons would come, you know there was a bit of hit and miss there and crops went, crops went belly up and people starved until the basic relationships were hit upon. You know, if I want to throw something up in the air and predict, you know, predict where it is going to come down or if I'm firing a cannon ball or something at someone and there's some, a guy called Newton described all that mathematics that we still use today and in fact we use to, that we use to take pot shots at the moon. I mean when they launched the astronauts to the moon I mean the rocket didn't go all the way there, it only went for the first few minutes, after that it was just a really good shot and that's, um, and I mean um exactly the same, you know like exactly the same set of assumptions, you know, the same underlying relationships rule everything. I mean there's not one piece of mathematics that works here on earth and another piece of mathematics on the moon, I mean it all works, if it works, it works everywhere and I know that the latest developments in quantum theory seem to be at odds with that, but quantum theory is an incomplete theory, so. But, yeah, I think it's about describing the real world, I mean you can describe the real world in gėneral terms with something like statistics or you can describe the real world in specific terms with something like, something like mathematical physics, or you can describe the, you can describe interactions in the real world, I mean psychological things such as, you know, people running businesses and trying to make money and behaviour on the stock market and that's also modelled pretty well mathematically.

Dave makes an obvious personal connection with mathematics, and Vitali expresses his view that mathematics is a language for dealing with the world, and a way of developing thought processes. Eddie talks of describing all aspects of the real world with mathematics: although he gives many individual models as examples, he is obviously focused on the overall idea of describing reality.

4. Discussion

The group of students interviewed seems to have overall quite sophisticated ideas about the nature of mathematics. This is not unexpected. They are specialising in the area: they have been studying mathematics all through high school, and for at least a year at tertiary level. Nevertheless, even in this group of students, there is a range of conceptions about their subject. Whereas they are all familiar with the notion of using mathematical techniques to describe aspects of the world around them, they show different levels of focus. At the narrowest level is a focus on mathematical components, at the middle level there is a focus on the mathematical models built and used to describe real life situations, and at the broadest level there is a focus on the process of modelling any aspect of reality and the way of thinking implied. The importance of modelling in mathematics, and its unifying nature in the world of professional mathematics has been noted by mathematicians themselves [**2**].

It is important to note that the conceptions are hierarchical, and that students who show evidence of the broader conceptions can also discuss mathematics using the narrower ones. George, for example, discusses the notion of financial models, but then also describes their components. Eddie speaks of mathematics describing the real world, but then follows up with specific examples of modelling situations. The converse is not true, however: Monique focuses on components, and despite questioning is unable to explain the broader use of these components. Thus, students who are aware of the broader conceptions can use the full range of ideas to further their understanding of the subject.

In terms of implications for teaching and learning, the most obvious lesson is that this wide range of conceptions exists in students studying mathematical sciences at university level. Moreover, the full range of conceptions was present in all years, implying that the categories are not developmental but simply a feature of students' experience. As mathematics educators, we need to cater for this variation in terms of our pedagogical methods and materials. Firstly, we can help students become aware of the range of variation in conceptions of mathematics. This may be enough in itself to encourage some students towards broader views. Any activity that allows students to explore the nature of their thinking and compare it to others' can form the basis of discussion. For instance, the usual discussion surrounding assessment tasks can be directed towards an exploration of the different ways that students may tackle such tasks. Research on using phenomenographic outcomes, such as those presented in this study, has shown that simple awareness of difference can be a catalyst for change [**11**]. Secondly, we can provide activities and assessment that encourage students towards the broadest levels of understanding of mathematics, and away from the narrowest levels. The following quotes from an earlier study with students in the same mathematics course [**10**] illustrate the difference:

> *Chris:* Lecturers create them [assignments] in such a way that you have to understand your work before you answer, because most of the assignments that we're given are actually real situations, they're not just made up scenarios.

> *Danny:* (Why would you want to rote learn things?) People do, and they do really well. (Well why is that?) Because if you are doing a lot of maths stuff and you have to reproduce proofs they just learn it all and write it all out.

(And you think that is superior to your attempt to understand the stuff?) No, it's not superior; I'd rather understand it, but you can get better marks for rote learning.

Learning materials that are set in real contexts, and that encourage students to investigate the mathematical approach and the role of mathematics in their professional and personal lives (for example, [**16, 18**]) will lead them towards the broadest conceptions of mathematics. Assessment that is based on learning and reproducing mathematical theory will tempt even those students with the broadest conceptions of mathematics towards the narrowest views. In summary, curriculum needs to accommodate variation in students' conceptions of mathematics, both because this variation exists and in order to help students broaden their views of their subject. It can also be important to explore the nature of work as a professional mathematical scientist and to demonstrate the applicability of students' studies to their future professional roles.

References

1. K. Crawford, S. Gordon, J. Nicholas, and M. Prosser, *Conceptions of mathematics and how it is learned: The perspectives of students entering university*, Learning and Instruction, **4** (1994), 331-345.
2. K. Houston, *Teaching modelling as a way of life*, Quaestiones Mathematicae, Supplement **1** (2001), 105–113.
3. S. Krantz, *How to teach mathematics*, (2^{nd} edition), American Mathematical Society, Providence, RI, 1999.
4. F. Marton, and S. Booth, *Learning and awareness*, Lawrence Erlbaum, NJ, 1997.
5. National Committee for Mathematics, *Mathematical sciences: Adding to Australia*, National Board of Employment, Education and Training, Canberra, 1996.
6. G. Perrett, L. N. Wood, and G. H. Smith, *Advanced mathematical discourse: A case study*, Literacy and Numeracy Studies, (2003), (in press).
7. P. Petocz, and A. Reid, *Relationships between students' experience of learning statistics and teaching statistics*, Statistics Education Research Journal, **2**(1) (2003), 39–53.
8. A. Reid, *Musicians' experience of the musical world: Relations with teaching and learning*, In C. Rust (Ed.), Improving Student Learning Through the Disciplines, Oxford Brookes, Oxford, 2000.
9. A. Reid, *Self and peer assessment in a course on instrumental pedagogy*, In D. Hunter and M. Russ (Eds.), Peer Learning in Music, (pp. 56–62), University of Ulster, Belfast, 2000.
10. A. Reid, and P. Petocz, *Students' conceptions of statistics: A phenomenographic study*, Journal of Statistics Education, **10**(2) (2002).
11. A. Reid, *Is there an 'ideal' approach to academic development?*, In A. Goody and D. Ingram (Eds.), Spheres of Influence: Ventures and Visions in Educational Development, University of Western Australia, 2002, online `www.csd.uwa.edu.au/ICED2002/publication/`.

12. A. H. Schoenfeld, *Purposes and methods of research in mathematics education*, In D. Holton (Ed.), The Teaching and Learning of Mathematics at University Level: An ICMI study (pp. 221–236), Kluwer, Dordrecht, 2001.
13. G. H. Smith and L. N. Wood, *Assessment of learning in university mathematics*, International Journal of Mathematical Education in Science and Technology, **31** (2000), 125–132.
14. K. Trigwell, and A. Reid, *Introduction: Work-based learning and the students' perspective*, Higher Education Research and Development, **17**(2) (1998), 141–154.
15. L. N. Wood, *Communicating mathematics across culture and time*, In H. Selin, (Ed.), Mathematics across cultures (pp. 1–12), Kluwer, Boston, 2000.
16. L. N. Wood, and G. Perrett, *Advanced mathematical discourse*, University of Technology, Sydney, 1997.
17. L. N. Wood, and P. Petocz, *Video in mathematics learning at the secondary-tertiary interface*, In W. Spunde, S. Cretchley and R. Hubbard, (Eds.) The Challenge of Diversity: Proceedings of the Δ 99 Symposium on Undergraduate Mathematics, Rockhampton, Queensland, 1999, online `http://www.sci.usq.edu.au/delta99`.
18. L. N. Wood, P. Petocz, and G. H. Smith, *Terror, tragedy and vibrations—Using mathematical models in engineering*, video, University of Technology, Sydney, 2000.
19. L. N. Wood, *The secondary-tertiary interface*, In D. Holton, (Ed.), The Teaching and Learning of Mathematics at University Level: An ICMI Study (pp. 87–98). Kluwer, Dordrecht, 2001.

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Preface

The Remarkable Delta03 conference is co-hosted by the Department of Mathematics and Statistics of the University of Otago and the Department of Mathematics and the Department of Statistics of the University of Auckland. Delta03 is backed by an International Steering Committee made up of representatives from the Southern Hemisphere countries of Argentina, Australia, New Zealand, South Africa, and Uruguay. This steering committee currently consists of: Derek Holton (New Zealand, Chair), Nestor Aguilera (Argentina), Pat Cretchley (Australia), Johann Engelbrecht (South Africa), Victor Martinez Luaces (Uruguay), Jan Persens (South Africa), Ivan Reilly (New Zealand) and Christina Varavsky (Australia).

This Conference is the fourth in a series of conferences on the undergraduate teaching and learning of Mathematics and Statistics, as part of a collaboration between Southern Hemisphere countries. The first conference, Delta97 (delta implying change) was held in Brisbane, Australia, in November 1997. The second conference, Delta99, took place in November 1999 at Laguna Quays, Queensland, Australia. The third conference, Warthog Delta01, was held in Kruger National Park, South Africa, in July 2001. The Delta conferences take place biennially, and the next one, Delta05, will probably be in Australia.

The theme for Delta97 was What Can We Do to Improve Learning?, for Delta99 it was The Challenge of Diversity, and for Delta01 it was Gearing for Flexibility. For Delta03 the theme is From all Angles.

The 120 delegates, attending from 12 different countries worldwide, are responsible for about 80 contributed presentations, panel discussions and round table discussions. The conference has two publications: the Proceedings, consisting of peer reviewed research papers, and the Communications (subject to editorial scrutiny) largely comprising reports on teaching experiences and research in progress.

We believe that the deliberations at this meeting will influence the course of future tertiary mathematics and statistics education worldwide. It is broadly accepted that skills and training in the quantitative sciences will be crucial to success in the future. Thus our belief is that this meeting will be one of the most important of 2003.

Our thanks to the group who worked on editing, compiling and producing this issue of the Proceedings, especially to, Mike Thomas, Greg Oates, Olita Moala, Betty Fong and Philip Sharp.

All papers in these Proceedings have been subject to the normal process of journal peer review, including the papers from the two plenary speakers who chose to submit a paper. In all 57 papers were submitted to the conference for consideration as full papers, and only the 22 that were accepted as full papers appear in this volume.

All opinions expressed herein are those of the authors themselves, and not necessarily of the Delta03 organising committee.

Derek Holton and Ivan Reilly
Co-Convenors
Remarkable Delta:03

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